# HILLSHADING WITH CONTOURS

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## ABSTRACT

Kitiro Tanaka, a Japanese cartographer, published two alternative contouring methods in 1932 and 1950. Inclined contour traces are defined by the intersection of topography with a series of evenly spaced, inclined planes, as opposed to the traditional horizontal planes used for most contours. Illuminated contours use black and white contours of varying thickness on a gray background to give the impression of an obliquely lit three-dimensional (3D) surface. Methods to automate and modify Tanaka's methods using Geographic Information System (GIS) technology are outlined here.

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## INTRODUCTION

Contour lines are a unique, useful and straightforward way to represent topography. They are by far the most quantitative of the cartographic techniques for representing continuous surfaces. The map user can make a direct determination of elevation, and easily calculate relief, slope and aspect. The mapmaker's task is also clearly defined when constructing contours. Given elevation values at known locations, (s)he interpolates values between data points and constructs a smooth curve based on these data. Furthermore, Geographic Information Systems (GIS) technology has automated this process considerably. Contours, however, are not without issue. One of the most common criticisms of contours is that map users, especially novices, have a difficult time visualizing topography from contours.

The two alternative methods of constructing contours discussed here are credited to the Japanese cartographer Kitiro Tanaka. The first is the inclined contour method, devised by Tanaka and published in 1932. The second is the illuminated contour method, published in English by Tanaka in 1950.

## INCLINED CONTOURS

Inclined contours are the traces that a series of parallel, evenly spaced, dipping planes would make when intersected with a continuous surface. The resulting traces in three dimensions are then projected orthogonally to the horizontal mapping plane (**Figure 1**). Kitiro Tanaka introduced this unique representation of topography in 1932 and dubbed his method the orthographic relief method. To illustrate the results, Tanaka included a map created with these new contours of Mt. Fuji, Japan. He used planes having an aspect direction to the south and a 45° slope. (**Figure 2**).

Such contour traces have come to be known as inclined contours, although some authors find the term misleading (Robinson and Thrower, 1957, Oberlander, 1968, Robinson and Thrower, 1969). As a form of shorthand, I will refer to contours created with Tanaka's orthographic relief method as inclined contours and those created by intersecting a surface with horizontal planes as horizontal contours.

The methodology for creating inclined contour maps by hand is straightforward but time consuming. Tanaka began with a topographic contour map, then drew parallel, evenly spaced lines on the map from west to east. The values of these parallel lines change at a constant rate and define the dip of the inclined planes. Topographic contours are then redrawn, beginning with the lowest topographic contour that intersects a given parallel line. As this parallel line intersects a topographic contour of higher elevation value, the inclined contour trace is transposed to the intersection of this topographic contour with the parallel contour to the north (**Figure 3**).

Inclined contours can help the map user visualize surfaces in two ways. First, individual inclined contour traces can be thought of as a hybrid between contours defined by horizontal planes and profiles defined by vertical planes. Unlike a perspective view of a series of profiles, however, each inclined contour is in its planimetrically correct position (Tanaka, 1932, Robinson and Thrower, 1957). Changing the planes' angle of inclination from horizontal will vary inclined contours from traces that resemble the original contours with angles near 0° to a series of nearly straight, parallel lines (vertical profiles in map view) with angles near 90°.

Second, inclined contours are a form of analytical hillshading with oblique illumination, which facilitates the visualization of surfaces. Analytical hillshading uses variations in shades of gray to give two-dimensional maps a three-dimensional appearance. Traditional contours derived from horizontal planes are difficult for most people to visualize. These dark contours on a light surface create tonal variations based on the density of traces. This trace density is related directly to the slope of the topography. The resulting tonal variations follow the rule "the steeper, the darker". The effect is similar to topography being illuminated from directly above, and is also known as slope shading (Imhof, 1982).

An example of the vertical illumination hillshading effect from horizontal contours is illustrated in **Figures 4**. The traces are contours of the Highland Mountains south of Butte, Montana resulting from intersecting the topography with horizontal planes spaced 20 meters apart. An example of a horizontal plane intersecting this topography is represented in **Figure 5**. The topography was created from a US Geological Survey Digital Elevation Model (DEM) with each grid cell of the DEM measuring approximately 30 meters square. Contours on **Figure 4** and all other maps in this article are not labeled to give a clear impression of the hillshading effect. The result is that the steepest slope, regardless of their aspect direction, are darkest. Although experienced map users may be able to easily visualize the topography, and any map user could determine ridges from valleys by querying contour values, most potential map users do not get a clear visual image of topography from such displays.

Tanaka's main objective of his inclined contour method was to quantitatively use the black contours on a white surface to emulate the tonal variations one would see from a matte reflector illuminated from an oblique angle. In his 1932 example, the 45° south dipping planes would be equivalent to obliquely illuminating the topography from the south, with a light source inclined 45° from horizontal. Imhof (1982) believed oblique illumination superior to vertical illumination in most cases, and stated that "slope shading [with vertical illumination] has been made obsolete today, its unrealistic framework of light and shade being difficult to appreciate and looking strangely unnatural" (p. 202).

One concern with this method is that south dipping inclined planes create the illusion of hillshading from the south. For many map users, this results in visualizing an inversion of topography, called the hypsographic effect. Most analytical hillshading uses illumination from the northwest or northeast to overcome this effect. Suggestions to overcoming this effect with inclined contours have ranged from rotating the map 135° clockwise to emulate northwest lighting (Tanaka, 1932), to adding additional shading between contours (Robinson and Thrower, 1957; Robinson, 1961) (**Figure 6**), to thickening inclined contour traces in areas requiring shading (Thrower, 1963).

Cartographers have automated the construction of inclined contours with GIS (Peucker et. al., 1975; Yoeli, 1976; Peucker, 1980). I use a simple technique suggested by Peucker et. al. (1975) to show traces resulting from intersecting the Highland Mountains with inclined planes dipping south at 45° (**Figure 7**) to create the inclined contour map of the area shown in **Figure 8**. The method, illustrated in **Figure 9**, begins with a the 30 meter DEM, creates a plane with the desired strike and dip over the same extent, adds the two grids together, and finally creates horizontal contours from the new surface. Adding an inclined plane to the original elevation values increases the overall relief in the area considerably. For this reason, the contour interval for the inclined contours increased to 50 meters from the 20 meter interval used for the horizontal contours. A version of **Figure 8** with additional oblique hillshading and layer tinting to represent changes in elevation is included as **Figure 10**. Such displays can be constructed with any GIS software package capable of manipulating and contouring grids. A step-by-step explanation of creating this display using ESRI ArcView GIS can be found at http://www.mbmg.mtech.edu/gis_hillshading.htm.

## ILLUMINATED CONTOURS

Tanaka's second method, the relief contour method, is commonly called the illuminated contour method. Tanaka used a medium gray background, assumed a northwest direction of illumination, then drew contours that would be illuminated with white ink and contours that would not be illuminated with black ink. This part of the method was not originated by Tanaka; such maps occurred as early as 1870 (Imhof, 1982). Tanaka refined the method by meticulously varying thickness of contours based on the cosine of the angle between the map direction of illumination and the aspect (direction of steepest slope) of the topography. His map of the area near Kagoshima in Kyushu, Japan is a striking visualization of a volcanic landscape (**Figure 11**).

Tanaka constructed this map by hand using a simple calligraphic technique. He filed his drawing pen to the maximum contour thickness desired and kept his pen oriented parallel to the direction of illumination. Resulting contours are thickest when perpendicular to the illumination direction and thinnest when parallel to the illumination direction. Everywhere contour thickness is proportional to the cosine of the angle $\square_T$ between the illumination direction ($I_A$) and the aspect direction of the contours (A) (**Figure 12**). Resulting tonal variations approximate patterns associated with analytical hillshading with oblique illumination.

The non-illuminated contours are analogous to shadows that would be cast if each contour corresponded to a step in an otherwise flat, layered topography. A related method, the shadowed contour method automated by Yoeli (1983) varies thickness of only non-illuminated, black contours on a white background (**Figure 13**). The illuminated contours can be thought of as photographic negatives of these shadowed contours on the illuminated side of a surface. Physical models of layered topography, such as models built from stacking cardboard cutouts of each contour interval, are similar in appearance to Tanaka's method when obliquely lit and viewed from above (**Figure 14**).

The stepped or terraced appearance of Tanaka's illuminated contours is the most common criticism of this method (Imhof, 1982). Part of the reason for the terraced appearance is that Tanaka varied thickness based solely on aspect of the surface and the direction of illumination (**Figure 12**). Aspect is the map direction that a surface element faces. Without knowing the slope of this surface element, however, one cannot specify its orientation in three dimensions. Alternatively, a 2.5-D surface element can be uniquely defined by one vector that is normal to the surface element.

Analytical hillshading is based on this surface normal vector and the illumination vector, both defined in three dimensional space. Most analytical hillshading uses the Lambertian assumption. This assumption states that apparent brightness of a matte surface will vary with the cosine of the angle □ between the illumination (I) and the surface normal (SN) vector. **Figure 15** shows a comparison of these two vectors with those used by Tanaka. This angle □ can vary from 0° to 90°. At 0°, the illumination vector is perpendicular to the surface resulting in a strongly illuminated white surface. At 90°, the illumination vector is parallel to a surface resulting in a non-illuminated black surface. Tanaka was aware of the simplification inherent in his method, but chose his approximation of analytical hillshading because it allowed the simple method for drawing by hand.

Several cartographers have automated Tanaka's procedure using the angle $\square_{T}$ with GIS (Peucker et. al., 1975; Peucker, 1980; Yoeli, 1976). Kennelly and Kimerling (2001) suggest an alternative GIS implementation using the angle □. **Figure 16** is an example using this modified Tanaka method for the Cascade Mountains of Washington State, including Mt. St. Helens and Mt. Adams. **Figure 17** illustrates the steps of this procedure for Mt. Adams. First, we created an analytical hillshading grid from the DEM, and reclassify the hillshading grid into the desired number of classes of □ (e.g. 18 classes of 5° for the 90° of □). We converted this reclassified grid into a GIS polygon coverage and intersect it with the contours created from the DEM. Finally, we assigned contours to black and white and adjust line thickness based on the classes derived from □. A step-by-step explanation of creating this display using ESRI ArcView GIS can be found at http://www.mbmg.mtech.edu/gis_hillshading.htm. A version of **Figure 16** using layer tinting to represent changes in elevation is included as **Figure 18**.

**Figure 16** results in a three dimensional rendering of a contour map similar to Tanaka' map in **Figure 11**. The main difference is that the thickness of contour traces varies with both changes of slope and aspect in **Figure 16**. This reduces the terraced appearance criticized with Tanaka's method. The disappearance of this stepped appearance is especially obvious in relatively flat areas. Because these areas have a low slope, contour thickness is minimized.

## CONCLUSIONS

Contours are both a quantitative and visual method for representing topography. Two alternative contour methods based in historical cartography are automated and modified with GIS technology. Inclined contours are defined by traces intermediate between contours derived from horizontal planes and vertical profiles derived from vertical planes. Illuminated contours give a strong impression of an obliquely illuminated three dimensional surface by varying thicknesses of black contours on non-illuminated topography and white contours on illuminated topography. Both methods result in generalized and stylized representations of continuous surfaces.

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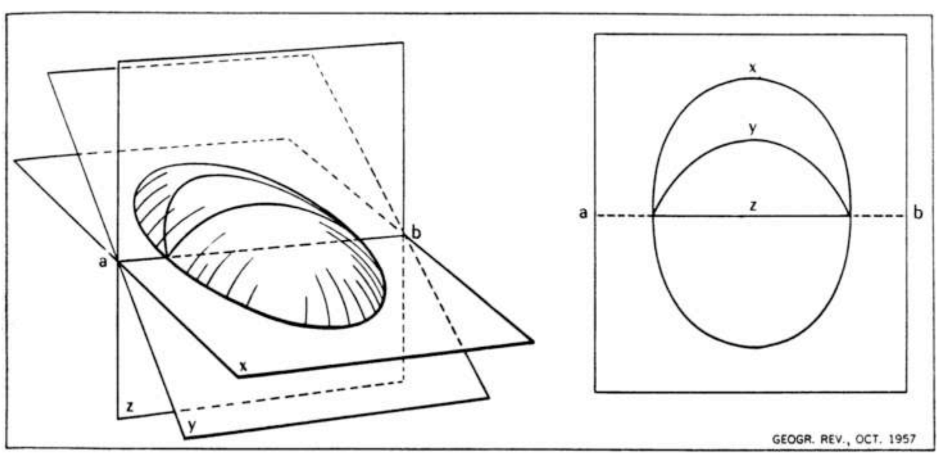


Figure 1. A horizontal plane (x), inclined plane (y), and vertical plane (z) intersecting a surface. The illustration on the left is a perspective view and the drawing on the right is a map view. The map view includes three traces resulting from the three planes intersecting topography. Trace x is a traditional contour, trace y is an inclined contour, and trace z is a profile. (Reprinted from Robinson and Thrower, 1957 (Figure 2 on page 513) with permission from the Geographical Review)

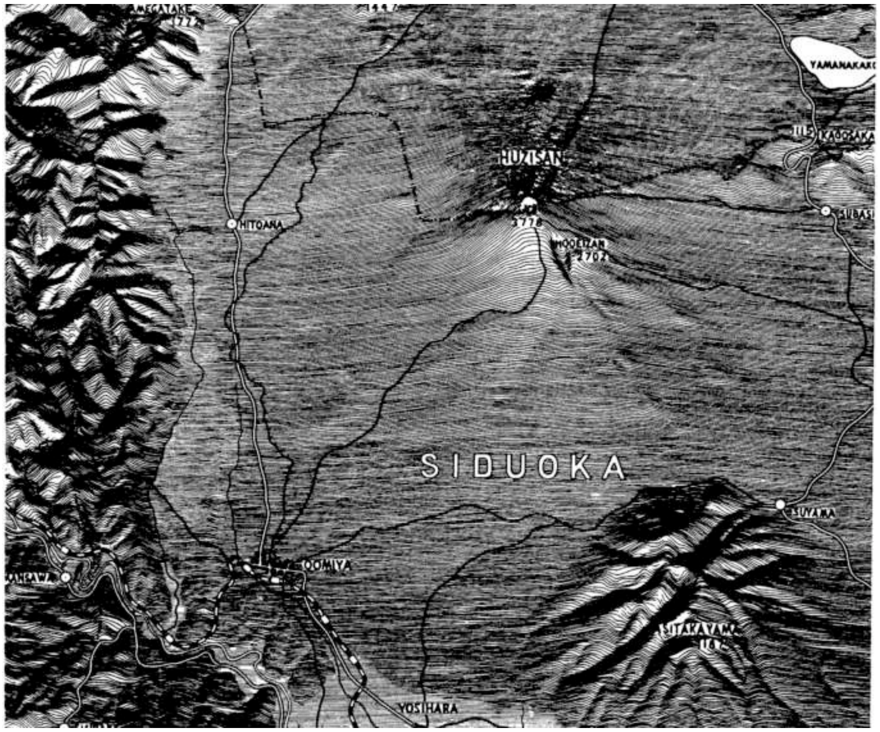


Figure 2. An inclined contour map of a portion of Mt. Fuji, Japan. The inclined plane used to define the traces had a strike of E-W and a dip of 45° to the south. (Reprinted from Tanaka, 1932 (Plate 2 on page 212) with permission from the Geographical Journal)

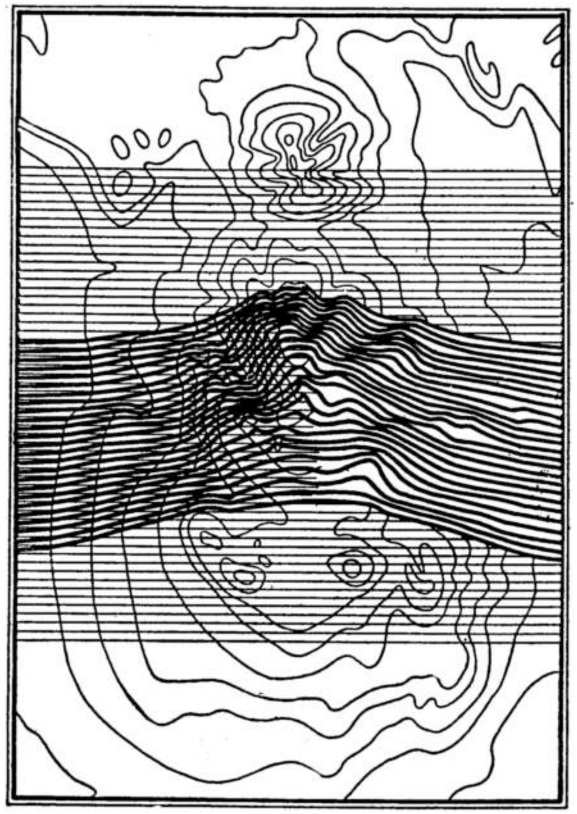

Figure 3. Construction of inclined contours by hand. The thin, straight, parallel lines are the inclined planes intersecting a horizontal surface. The intermediate closed curves are the original contours defined with a horizontal plane. The thick, wavy lines are the inclined contours, traces of the inclined planes intersecting the topography and projected orthogonally onto the map. The first two sets of lines have been removed from an area on the right of the map to show final results. (Reprinted from Tanaka, 1932 (Figure 2 on page 216), with permission from the Geographical Journal).

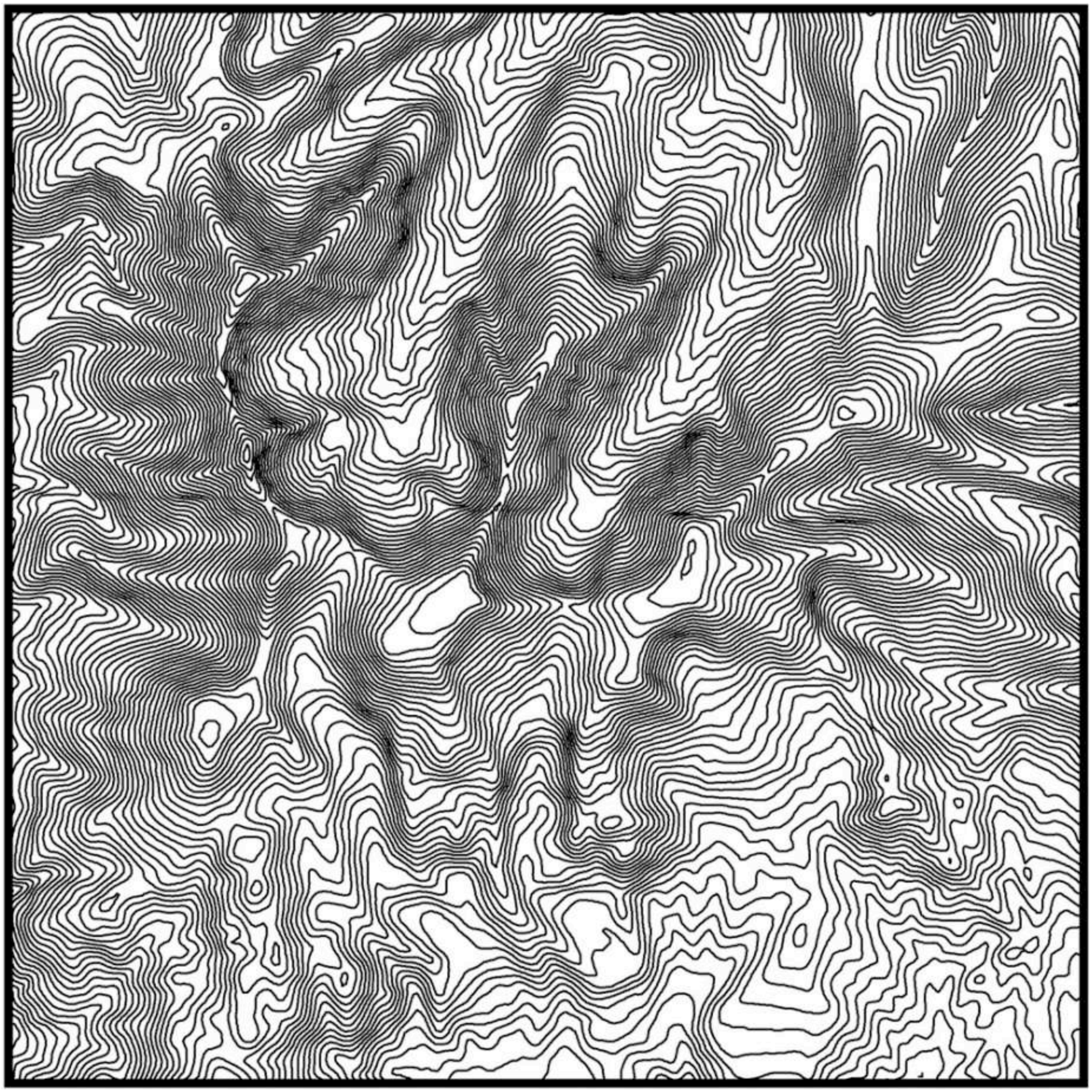

Figure 4. A map generated with GIS of contours from horizontal planes intersecting the Highland Mountains of Montana created from a Digital Elevation Model (DEM). The contour interval is 20 meters. The map scale is approximately 1:100,000 and north is to the top of the page. (Reprinted from Kennelly, 2002 (Figure 4 on page 431), with permission from the Journal of Geoscience Education).

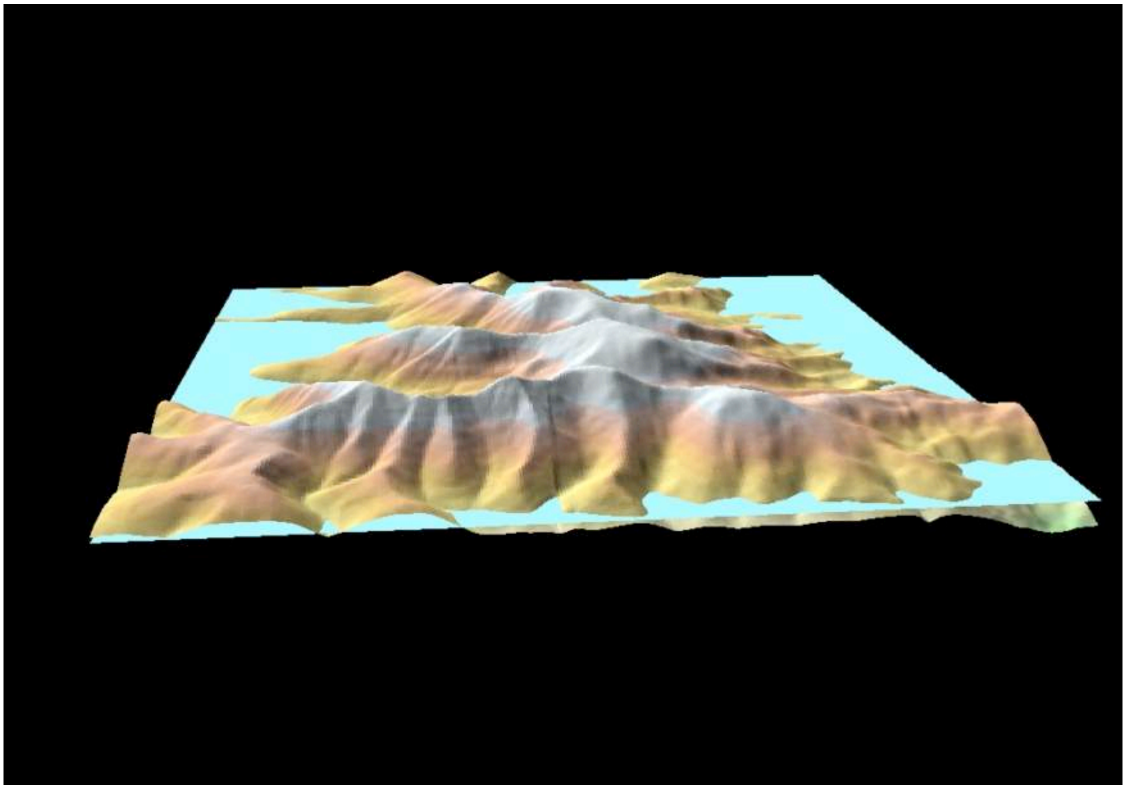

Figure 5. A 3D representation of a horizontal plane  intersecting the topography of the Highland Mountains south of Butte, MT.  The traces from a number of such evenly spaced planes would create the contour map shown in Figure 4.

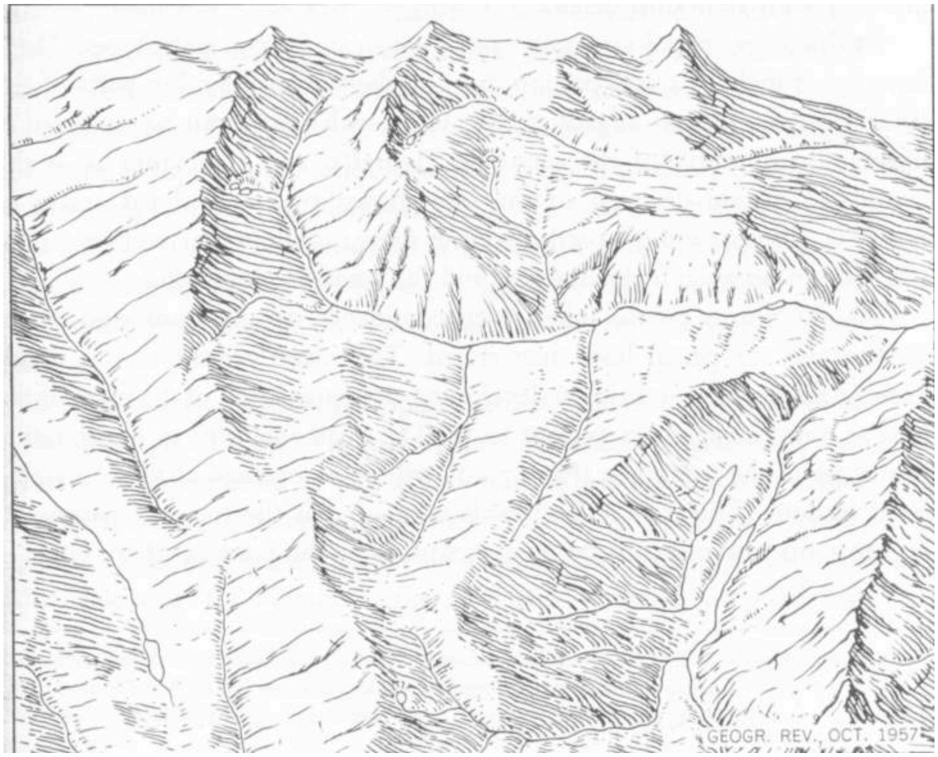


Figure 6. A map created by Arthur Robinson and Norman Thrower of the Camp Hale area, CO using the inclined contour method. The traces that resemble profiles are based on inclined contours. Additional shading is added to create an illumination effect from the northwest. Inclined contours and hillshading are both planimetrically correct. (Reprinted from Robinson and Thrower, 1957 (Figure 10 on page 519) with the permission of the Geographical Review).

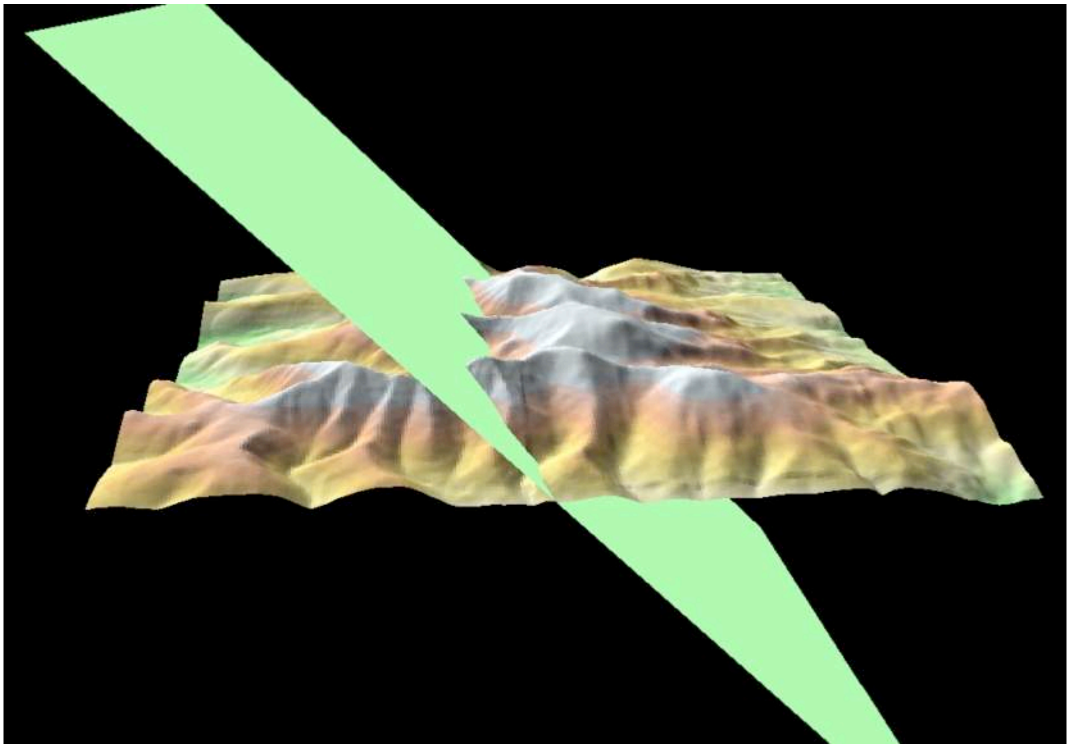

Figure 7. A 3D representation of an inclined plane dipping 45° to the south intersecting the topography of the Highland Mountains south of Butte, MT. The traces from a number of such evenly spaced planes would create the inclined contour map shown in Figure 8 if they are projected orthogonally onto a horizontal plane.

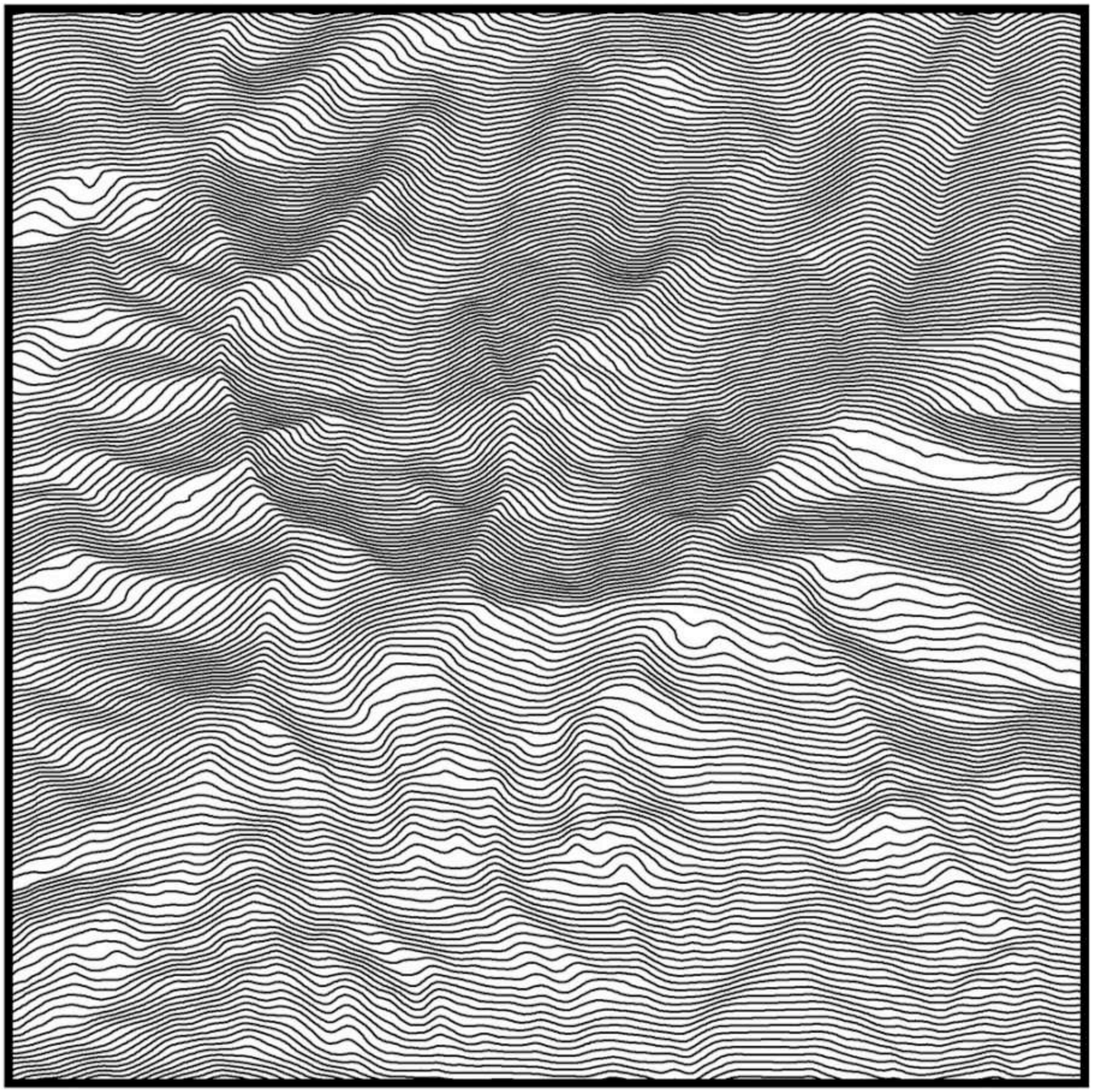

Figure 8. A map generated with GIS of contours from inclined planes intersecting the Highland Mountains of Montana created from a Digital Elevation Model (DEM).  The inclined plane used to define the traces had a strike of E-W and a dip of 45° to the south (See Figure 7). Inclined traces have a profile-like appearance with the page rightside up.  Inclined traces provide tonal variations that show topography with a more three-dimensional effect with the page upside down. The contour interval of the inclined surface is 50 meters. The map scale is approximately 1:100,000 and north is to the top of the page. (Reprinted from Kennelly, 2002 (Figure 5 on page 432), with permission from the Journal of Geoscience Education).

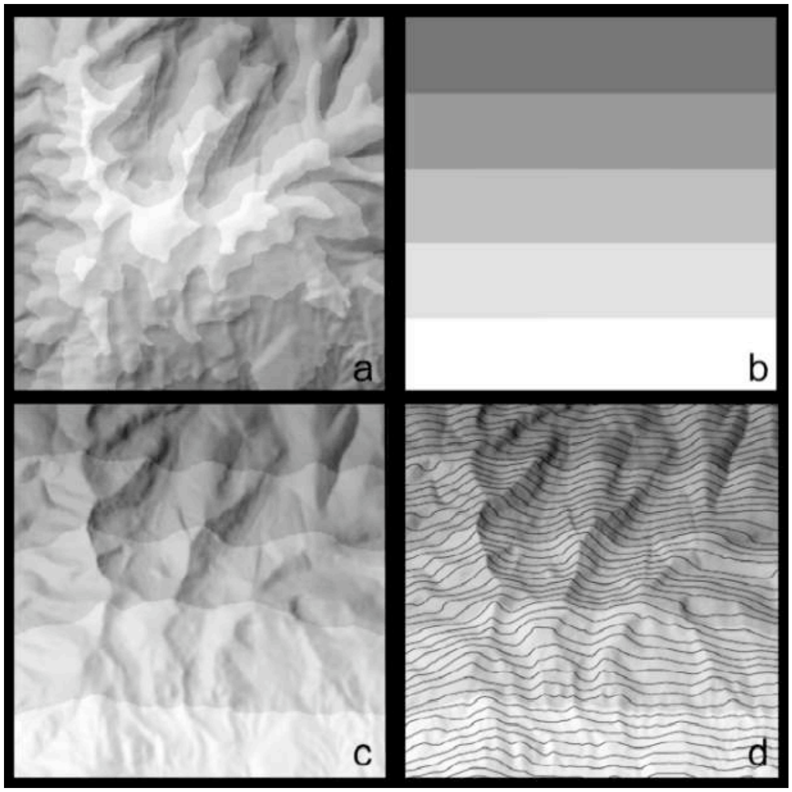


Figure 9. The GIS method used to construct the inclined contours of Figure 8. Lighter shades of gray represent higher elevations in all displays. a) The process begins with a DEM of topography. The DEM consists of a regular grid, with each grid cell having one elevation value. b) An inclined plane is created over the same area. The GIS is used to create a coverage of four points, one for each corner of the DEM. The corners are assigned elevation values to define an east-west striking, north dipping plane. These points are converted into a grid with the same cell size as the DEM c) The inclined plane grid is added to the topography grid. d) The new surface is contoured in the conventional manner. (Reprinted from Kennelly, 2002 (Figure 6 on page 432), with permission from the Journal of Geoscience Education).

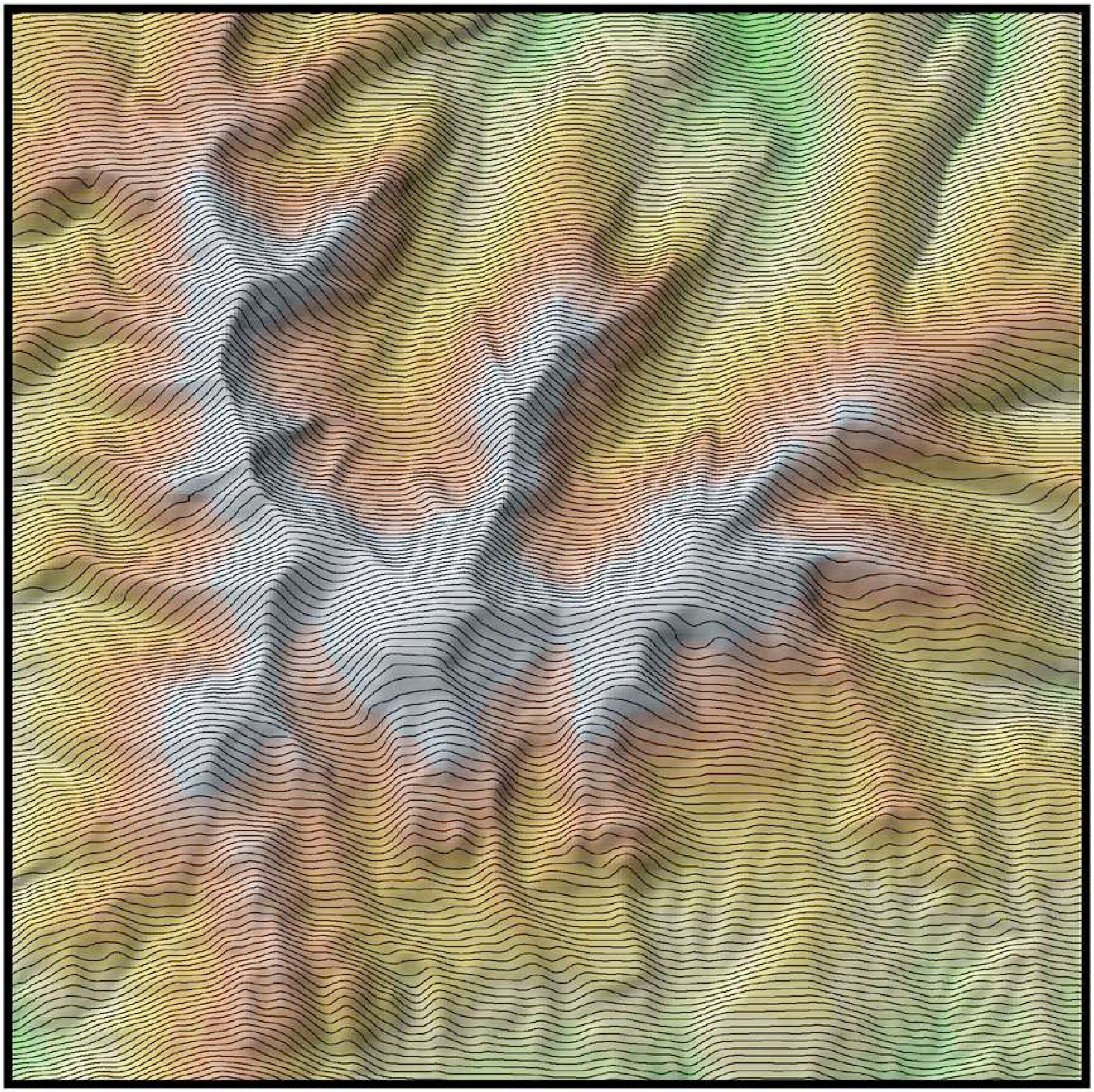

Figure 10. A color version of the inclined contour map in Figure 8, with layer tinting used to display elevation.

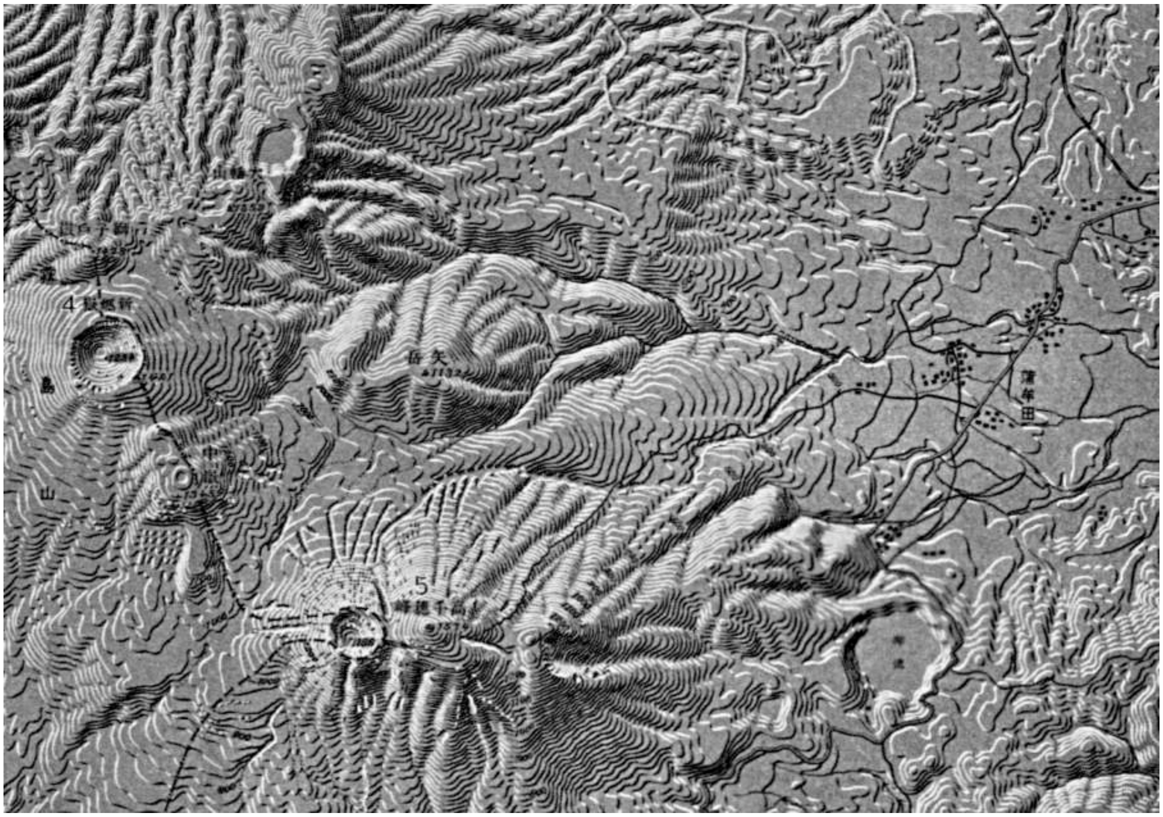


Figure 11. A portion of Tanaka's illuminated contour map of the Kirishima volcanic group near Kagoshima in Kyushu, Japan. (Reprinted from Tanaka, 1950 (Figure 7 on page 451). Reprinted with permission from the Geographical Review).

Figure 12. Azimuthal illumination ($I_A$) and aspect (A) vectors used to determine the thickness of contours with the Tanaka method. The thickness is proportional to the cosine of the angle $\square_T$ (Modified from Tanaka, 1950 (Figure 2 on page 445). Reprinted from Kennelly and Kimerling, 2001 (Figure 1 on page 112 ) with permission from Cartography and Geographic Information Science).

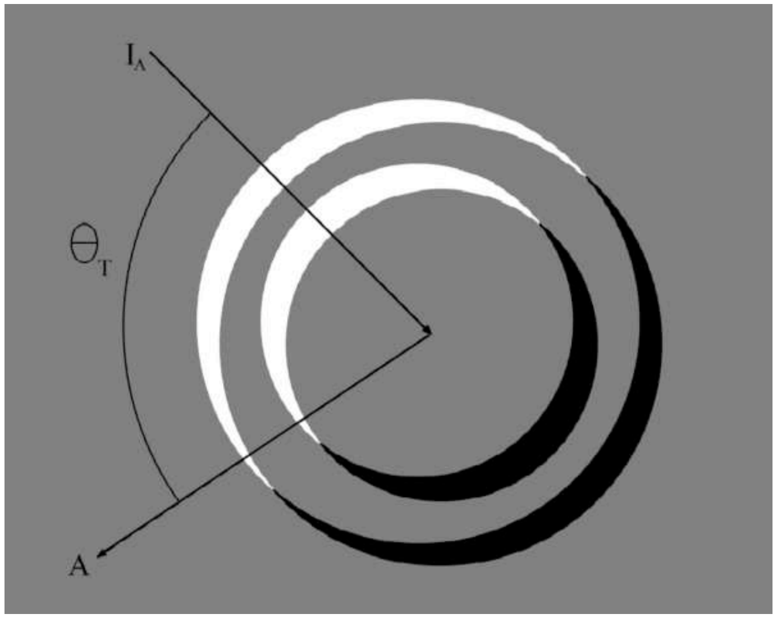


Figure 13. A portion of a computer generated shadowed contour map south of Haifa in Israel. (Reprinted from Yoeli, 1983 (Figure 18 on page 109) with permission of The American Cartographer).

Figure 14. A physical model of the Deer Lodge and Silver Bow Valleys near Butte, MT. Todd Trigsted created this model for the Montana Water Company by stacking cardboard cutouts based on topographic contours. (Courtesy of Todd Trigsted).

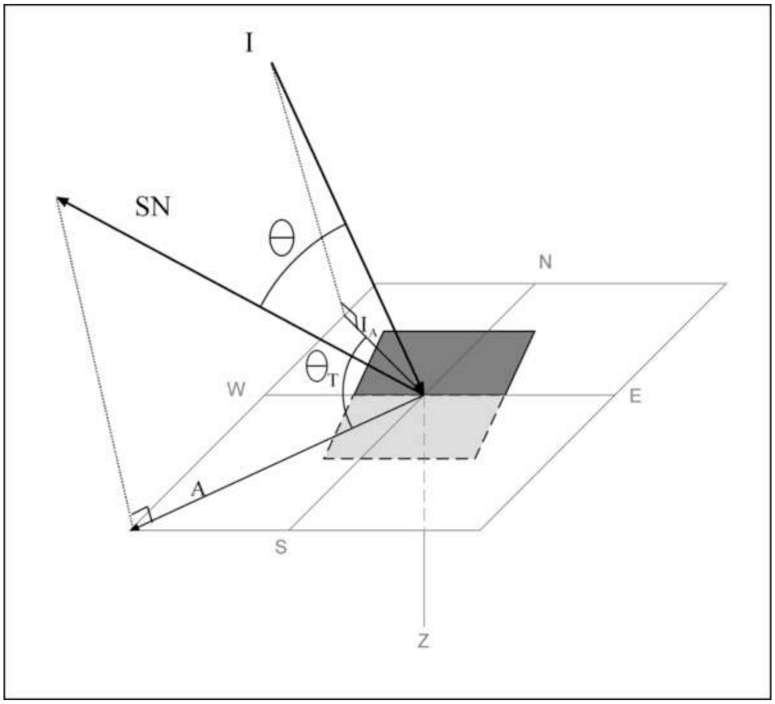


Figure 15. Illumination (I) and surface normal (SN) vectors in three dimensions used to determine the thickness of contours in this study. Line thickness is proportional to the cosine of twice the angle $\theta$. Vertical projections of vectors I and SN onto a horizontal plane result in Tanaka's vectors $I_A$ and A respectively (Compare with Figure 12). (Reprinted from Kennelly and Kimerling, 2001 (Figure 3 on page 113) with permission from Cartography and Geographic Information Science)

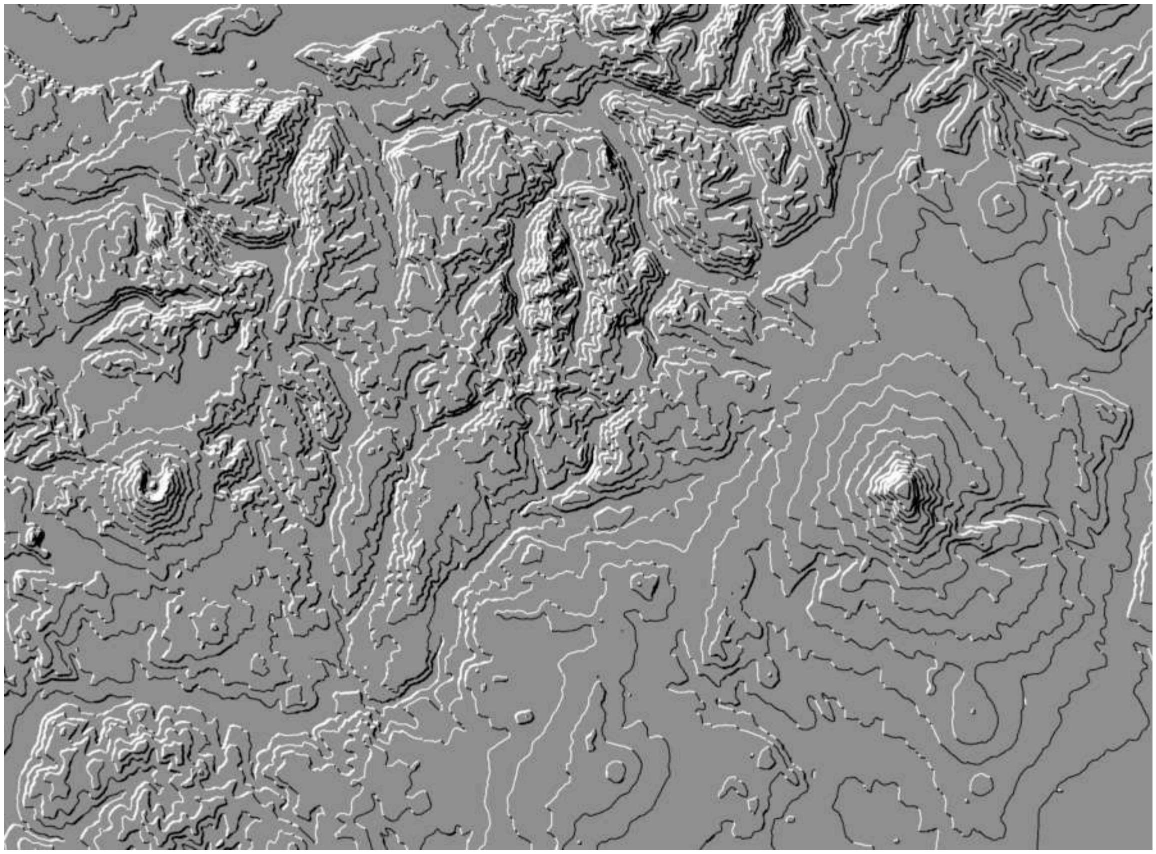

Figure 16. Illuminated contours the Cascade Mountains of Washington State. Peaks include Mt. Adams to the east and Mt. St. Helens to the west. The contour interval is 200 meters. Black vs. white contours and thickness were created with Kennelly and Kimerling's (2001) modified Tanaka method, based on the angle between the illumination vector and the surface normal vector. The scale is approximately 1:500,000 and north is to the top of the page. (Reprinted from Kennelly and Kimerling, 2001 (Figure 7 on page 117) with permission from Cartography and Geographic Information Science)

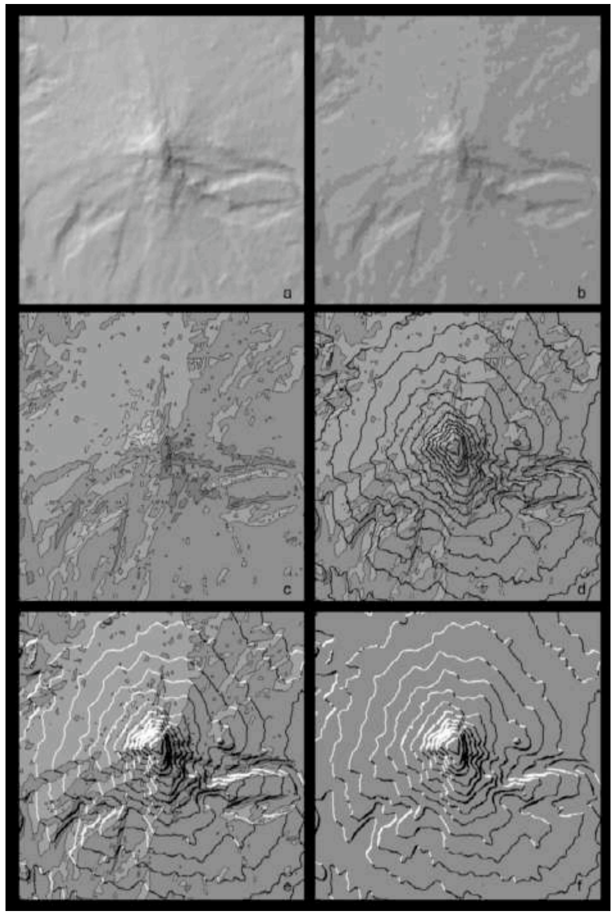


Figure 17.  The GIS method used to create the illuminated contours in Figure 16:  a) A hillshading grid is generated from the DEM. b) The hillshading grid is reclassified into a new grid with 18 classes, each representing a interval of 5° for the angle □. c) The reclassified grid is converted into a GIS polygon coverage.  d) Contours are generated from the original DEM and intersected with the reclassified polygon coverage.  This transfers hillshading information to each line segment that make up the contours. e) Black and white colors and appropriate thickness are assigned to each contour segment based on hillshading information. f) A medium gray background is applied for visual contrast and clarity. (Reprinted from Kennelly, 2002 (Figure 12 on page 435), with permission from the Journal of Geoscience Education).

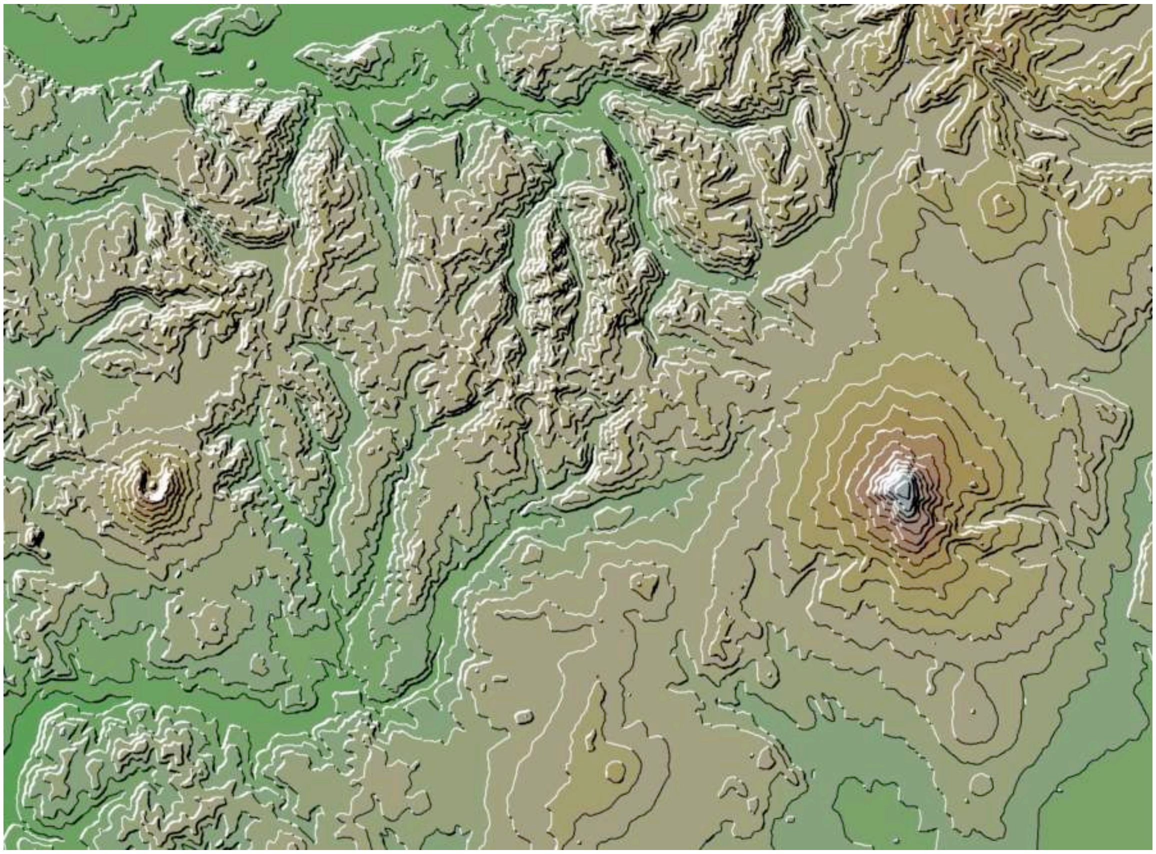

Figure 18. A color version of the illuminated contour map in Figure 17, with layer tinting used to display elevation. (Modified from Kennelly and Kimerling, 2001 (Journal cover), with permission from the Cartography and Geographic Information Science).