Abstract: Relief shading gives to a map not only aesthetic dimension, but it can also be used as information supplement in the map. An overview of existing shades of the mountain Medvednica near Zagreb, Croatia is given. Possibilities in modifying analytical shades and results in aesthetic dimension and perception of the relief are explored.

INTRODUCTION

Only few capital cities have the advantage of being so close to nature parks like Zagreb to the Medvednica Mountain does. Medvednica’s base can be easily reached from different areas of Zagreb by tram or bus, so citizens are traditionally oriented to Medvednica. They are passionate mountaineers; there are 47 mountaineering associations, as well as 8 mountain tenements on Medvednica [URL 1]. They are also very keen on sports: Zagreb organised the FIS World CUP race “Snow Queen Trophy 2006” which gathered around 20 000 spectators. Knowing all that, we can conclude that no Croatian mountain has so many cartographic representations like Medvednica does.

We picked three maps for illustration from a variety of existing maps of Medvednica (Fig. 1, 2 and 3).
Fig. 2: Panoramic view of Medvednica and Zagreb [URL 2]

Fig. 3: Bicycle map of the Medvednica Mountain [URL 2]
The main point of our interest on these maps is displaying relief by shading. On Fig. 1 and 2 shading is made by classical cartographic methods, while Fig. 3 shows analytical relief shading.

Impression of depth in visual display is a sophisticated process which is particularly important in cartography when we are faced with the problem of simulating three-dimensional information in a two-dimensional display.

Relief shading was developed by cartographers as an important technique for improvement of visual quality of maps, particularly in pointing relief differences between hilly and mountainous areas. Relief shading is a strictly specialised drawing technique in classical cartography. Talented artists, on the basis of contour system (and even aerial images), are visualising relief and hand drawing lightened and shadowed surfaces under an imagined light source (Fig. 4) (Frangeš 1998).

The main difference between computerized and manual relief shading is in the light direction. In classical approach, relief shading directions of the light rays are different by azimuth and by elevation, so shades are adapting to different shapes of models more easily. Differently oriented relief shapes can always be illuminated vertically onto stretching direction. Naturally, this shading method is slow and expensive; therefore there is an aspiration for automation of this process (Poslončec-Petrič, 2002).

As computing technology developed and digital relief models became available to more users, methods of automatic relief shadings were developed. Computerised relief shading by mathematic models are different from classical shading. Beside that, computerised shading is based on photometrical function, while classical approach, with fulfilling series of cartographic conditions, is still based on cartographer's subjective representation of experience.
Comparison of mathematical shading models and classical shading could be done by using rules of the so-called classic Swiss school (Imhof, 1965):

1. In classical shading, relief must be illuminated from the North side. Although this is opposite of experience from nature, we avoid the inverse effect (conversed perception hills - valleys).

2. In classical relief shading, own shadows are displayed, but not thrown shadows.

3. Soil type and vegetation do not affect shadow's intensity, but they do affect reflection ability of soil.

4. In classical relief shading, illumination on the same map could be changed by different azimuth and elevation due to more plastic display of the terrain.

5. Acquired relief shades could be illuminated and tarnished by needs, depending on the purpose of the map and reproduction method.

6. In classical relief shading, there is more contrast between illuminated and tarnished areas on higher altitudes. It may result with blurred valleys and sharpened peaks.

7. Plains, flat parts of terrain must be brighter or white even if the light source is not in the zenith.

A fair mathematical model of cartographic shading must fulfil all of these conditions. One of the methods is Brassel's analytic performance of Swiss school (Brassell, 1974). Analytic formulation and overview of different suggested procedures for analytic shading was made by Horn (1981).

MOTIVATION

Maps primarily showing relief may use shades along with hypsometry as basic map elements. In these cases, shades can be presented with more details using a wide scale of tones. Other maps (i.e. thematic and topographic maps) use shades as a secondary element that helps the perception of relief. In that situation, the shades leave the domination to the other cartographic signatures and the measure of weight (how much shades affect legibility of other elements on the map) becomes important. Special care to this dimension of shades should be given (Horn, 1981). The goal is to accomplish the best relief perception possible, to improve map aesthetics and to preserve legibility of other map elements using minimal weight of shades on the map.

The work of Castner and Wheate (1979) shows that using the shades on the topographic maps gives faster interpretation of the relief and serves as organization basis for all map elements. The researches of DeLucia (1972) shows that lower weight of shades on the map improve the correct interpretation of the other map elements and also shorten the time for interpretation. From this, we can conclude that using shades on thematic and topographic maps can improve the quality of the overall interpretation but very dominant shades can have the opposite effect on non-relief elements.

Based on Brassel's research (1974), this article investigates modifications of the light azimuth, height and intensity. Considering that in the Croatian cartography flat areas are traditionally presented without tone this was of special consideration.

The procedure described in this article was developed using the GRASS 6.0 GIS software and the results are implemented in script command r.modif.shades that can be downloaded from [URL 3]. Inside the script, descriptions of each parameter are given and other possibilities of presented modifications using this script can be explored.
METHODOLOGY

Let the shaded relief be the $R_{mn}$ raster matrix with the values in the range from 0 to 255. A lower value represents higher tone value. Assuming a linear relationship between matrix values and weight, with condition that zero value represents 100% of tone value and 255 represents 0% of tone value; the total weight of the shades can be expressed with equation:

$$W = \frac{\sum_{i=0}^{m-1} \sum_{j=0}^{n-1} \left(1 - \frac{R_{ij}}{255}\right)}{m \cdot n} \cdot 100\%$$

(1)

Other methods for estimation of the weight of shades are not found in available literature. Therefore, this simple formula will be used for the further examples. This equation does not consider the sensitivity of the human eye to light intensity. The weights calculated with this equation can be used for relative comparison of shaded relief, but these values do not represent the final weight of shades on the map because it mainly depends on the chosen range of tonal values.

MODIFICATIONS OF DIFFUSE SHADING

Diffuse shading is based on the Lambert cosine law. It is expressed by equation (Yoéli, 1965):

$$D_{ij} = \vec{t}_{ij} \cdot \vec{r}_{ij} \mid \vec{t}_{ij} \mid \mid \vec{r}_{ij} \mid , \quad i = 0,1,...,m, \quad j = 0,1,...,n$$

where $\vec{t}_{ij}$ is vector of light (often it is a constant vector) and $\vec{r}_{ij}$ is normal vector of relief surface. Function $D_{ij}$ is in interval $[-1, 1]$. Linear transformation from interval $[\min(D_{mn}), \max(D_{mn})]$ to interval $[0,255]$ is:

$$R_{ij} = \frac{255}{\max(D_{mn}) - \min(D_{mn})} \left( D_{ij} - \min(D_{mn}) \right).$$

This way the range of values of cosine of an angle is transformed into an 8 bit scale of tonal values, one which is used in computer visualisation. Now the weight of shades can be calculated by equation (1).

Vector of light $\vec{t}_{ij}$ in 3-dimensional space is unambiguously determined by three parameters, i.e. with a triple defining the radius vector $\vec{t}_{ij} = x_{ij}\vec{i} + y_{ij}\vec{j} + z_{ij}\vec{k}$. In analytical relief shading, this vector is often determined with parameters of height above the horizon $H_{ij}$, azimuth $A_{ij}$ and length $I_{ij}$. The height is the angle of the vector to plane $x\ y$; the azimuth is angle in plane $x\ y$ from conventional direction. These two ways for expressing the vector of light are connected by equations

$$H_{ij} = \arctan \left( \frac{z_{ij}}{\sqrt{x_{ij}^2 + y_{ij}^2}} \right), \quad A_{ij} = \arctan \left( \frac{y_{ij}}{x_{ij}} \right), \quad I_{ij} = \sqrt{x_{ij}^2 + y_{ij}^2 + z_{ij}^2}.$$  

Azimuth $A_{ij}$ should be measured from conventional direction. If the vector of light is constant for the whole relief model, then the third parameter (the length of vector) is not of importance and its value may be assumed to be 1.
If we allow to change the vector of light by its three parameters over the relief, \( \vec{I}_j = f(\vec{r}_j) \neq const. \), then the modifications of shades can be obtained.

On the relief model of the mountain Medvednica we will explore some modifications of these parameters and their influence on shades. The height scale in model is 4:1.

**Standard Diffuse Shading**

First, we will apply constant light to relief model of Medvednica with height \( H_j = 45^\circ = const. \) and azimuth \( A_j = 135^\circ = const. \) (azimuth is measured clockwise from the direction of North). The result is represented on Fig. 5. The weight of these shades calculated by equation (1) is 27%. The azimuth of light is chosen to put Northwest part of mountain in shadow. This part of mountain is steeper and colder in reality.

The advantages of the shades in Fig. 5 are a great deal of details, soft transition of tone values, and the main layout of the mountain is well presented. Poor emphasis of ridges parallel with light azimuth and relatively high weight factor can be considered disadvantages.
Modification of the Light Azimuth

By modifying the azimuth, the goal is to better depict landforms in all directions. Here, the azimuth modification is performed by following procedure:

1. First the aspect \( a_y \) of the model is calculated (angle between projection of surface normal to \( xy \) plane and conventional direction). The starting direction for aspect is chosen light azimuth \( A_y = A_{const.} \) (in this example \( A_y = A_{const.} = 135^\circ \)) and is measured clockwise.

2. Aspect \( a_y \) is transformed to one quadrant (values in range \([0^\circ, 90^\circ]\)) using equation
   \[
   a_y = \arcsin\left|\sin(a_y)\right|.
   \]

3. Transformed aspect is added or subtracted from chosen main azimuth direction \( A_y = A_{const.} \pm a_y \).

Fig. 6 shows shades of relief model of Medvednica with azimuth of the light \( A_y = A_{const.} + a_y \) and height \( H_y = 45^\circ = const. \) The weight of these shades is 28%.

The advantages of the shades on Fig. 6 are well emphasized ridges, the main trend of landform is still well presented and the contrast is higher. The presentation of lower landforms benefits the most from such azimuth modification. The landforms in West-east direction are presented poorly. Overall, it can be concluded that this
azimuth modification gives lot of advantages while still preserving the same weight. The only disadvantage is the loss of details.

Modification of the Light Height

In cartographic practice, it is often desirable to have shades which have white flat areas, that is, without tone value. Although this way we lose some perception of relief, the weight of shades on the map will be lower.

The next modification uses height of the light as function of slope of the relief $S_\phi$ expressed by equation $H_\phi = 90^\circ - S_\phi$, instead of constant height of 45°. This way the height of the light is perpendicular to the slope of the relief. Fig. 7 shows this modification, along with the azimuth modification. The weight of shades is 14%.

![Fig. 7: Shades of the relief model of Medvednica with modification of light height and azimuth](image)

Shades on Fig. 7 have white tone for flat areas, while the rest of shades are mostly preserved. The loss is on the perception of the relief but the weight is halved.
Modification of the Light Vector Length

The last parameter of light vector is its length which can be interpreted as light intensity. Sometimes it is desirable to have higher contrast in high parts of relief and lower contrast in lower areas. This can be accomplished by modification of length of vector of light in a way that for the lowest height in model \( I(\min(R_{mn})) = 1 \) and for the highest point in model \( I(\max(R_{mn})) = k \). The equation which gives linear relationship between height and intensity \( I_y \) is

\[
I_y = \frac{k - 1}{\max(R_{mn}) - \min(R_{mn})} \left( R_y - \min(R_{mn}) \right) + 1.
\]

If it is needed to have white flat areas after such modification then we have to read value \( D_{flat} \) in flat areas and perform transformation to range [0,255] by equation

\[
R_y = \frac{255}{D_{flat} - \min(D_{mn})} (D_y - \min(D_{mn})), \text{ for } R_y > 255, \; R_y = 255.
\]

Fig. 8 shows the shades with all three modifications (azimuth, height and intensity). The factor \( k \) for intensity is 2. The weight of these shades is 10%. 

Fig. 8: Shades of the relief model of Medvednica with modification of light height, azimuth and intensity \((k = 2)\)
CONCLUSION

Using the light vector as a function of the relief model instead of constant vector, different results in shades can be accomplished. Described modifications are simple, easy to interpret and can be applied to any relief model. These modifications do not necessarily lead to the final result, but may be used as a starting point for other modifications by which analytical shades are transformed into ones according to cartographic rules.

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